

Weak phase information from the color suppressed $\bar{B}_d^0 \rightarrow \bar{D}^{*0} \bar{K}^{*0}$ modes

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Abstract

The decay channels $\bar{B}_d^0 \rightarrow \bar{D}^0 \bar{K}^{*0}$, $\bar{D}^{*0} \bar{K}^{*0}$ are investigated for extracting weak CKM phase information. These channels are described by color-suppressed tree diagrams only and are free from penguin contributions. The branching ratios for these channels are found to be $\sim \mathcal{O}(10^{-5} - 10^{-6})$ which can be measured at the currently running B factories. The method presented here may be well-suited to determine the CKM angle γ .

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After the discovery of CP violation in K systems in 1964, there was a lot of enthusiasm to look for CP violation in other systems as well. CP violation is found to be rather small in K systems, however, theoretical predictions have suggested that the B system may be an ideal place for detecting sufficiently large CP violating effects. In fact large CP violation in B system has been observed in the currently running dedicated B factories such as BaBar at SLAC and BELLE at KEK. Over the years, a lot of work have been done in the area of CP violation with many facilities collecting data on B events. In the standard model (SM), CP violation arises from the non-zero weak phase in the complex Cabibbo-Kobayashi-Maskawa (CKM) matrix which is responsible for the charged-current weak interaction and is characterized by the so-called unitarity triangle. It is, therefore, imperative that the three angles of this triangle, termed as α, β and γ , be measured independently to get decisive information on the origin of CP violation.

Usually, these angles are extracted from CP violating rate asymmetries in B decays. The angle β (or $\sin 2\beta$) has been cleanly determined from the measurement of the time dependent CP asymmetry in the golden decay mode $B_d^0 \rightarrow J/\psi K_S$ [1]. The angle α can be measured using the CP asymmetries in $B_d^0 \rightarrow \pi^+\pi^-$ [2], but due to the existence of penguin diagrams there are theoretical hadronic uncertainties which are very difficult to quantify. The last angle which is hoped to be determined cleanly is $\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$. There have been many attempts, suggestions and discussions to measure this angle as cleanly as possible. The Gronau, London and Wyler (GLW) [3] method, which is usually termed as the golden method to extract γ , employs the $B \rightarrow DK$ modes to extract this angle. In the GLW method the interference of two amplitudes ($b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$) is being used which endows us to determine the angle γ by measuring the decay rates for $B \rightarrow D^0K, B \rightarrow \bar{D}^0K$ and $B \rightarrow D_{CP}^0K$ and their corresponding CP conjugate modes. However, it is experimentally difficult to measure the mode $B^- \rightarrow \bar{D}^0K^-$. As pointed out in earlier studies, the reason is that the final \bar{D}^0 meson should be identified using $\bar{D}^0 \rightarrow K^+\pi^-$, but it is difficult to distinguish it from the doubly Cabibbo suppressed $D^0 \rightarrow K^+\pi^-$.

There exist many studies in the literature [4] to help overcome the difficulty and to provide improved ways to determine the angle γ . It is highly desirable to have independent measurement of the angle γ (or otherwise the angle α), at least to the precision of the angle β as of today, to understand better the CKM mechanism of CP violation under the framework of the SM. But so far we have not been able to succeed in this effort. Given the various

methods, and wide range of options available, the measurement of the angle γ seems to be a better option. This is currently being done and will also be taken up in the second generation experiments. There is also another parameter, namely, $2\beta + \gamma$ which is also discussed in the literature [5] to be measured. Since β is well measured by now, therefore, the measurement of $2\beta + \gamma$ will be very much useful for the clean determination of γ . It should be noted here that we should measure the angle γ in all possible way (and as cleanly as possible) to independently verify the measurements, improve the statistics and to help resolve discrete ambiguities. To this end, we intend to present here another important and simple way to extract the weak phase $\gamma/(2\beta + \gamma)$ from the decay modes $\bar{B}_d^- \rightarrow \bar{D}^0 \bar{K}^{*0}, \bar{D}^{*0} \bar{K}^{*0}$.

In this paper, we consider the color-suppressed decay modes $\bar{B}_d^- \rightarrow \bar{D}^0(\bar{D}^{*0}) \bar{K}^{*0}$, to extract the CKM phase information. Several studies have been carried out using these decay modes [6, 7, 8] for the extraction of the angle γ . We, in this investigation, present another alternative method, which is very clean and simple to extract the weak phase $\tan^2 \gamma$ ($\tan^2(2\beta + \gamma)$). Since, as emphasized before, one needs to have as many clean methods as possible to improve the sensitivity and to resolve the discrete ambiguities the present method will be very much helpful in that direction.

First let us consider the decay channels $B^0 \rightarrow D^0 K^{*0}$ and $B^0 \rightarrow \bar{D}^0 K^{*0}$. It has been shown in Ref. [7] that the CKM angle γ can be determined by measuring the following six decay rates: $B^0 \rightarrow D^0 K^{*0}$ and $B^0 \rightarrow \bar{D}^0 K^{*0}$, $B \rightarrow D_{\text{CP}} K^{*0}$ (where $D_{\text{CP}} = (D^0 + \bar{D}^0)/\sqrt{2}$, is the CP even eigenstate of neutral D meson) and the corresponding conjugate processes. The $D^0(\bar{D}^0)$ meson is considered to decay subsequently to the flavor state $K^+\pi^-$ for which the ratio of the two amplitudes is found to be very tiny i.e., $r_D = |A(B^0 \rightarrow K^+\pi^-)/A(\bar{D}^0 \rightarrow K^+\pi^-)| = 0.06 \pm 0.003$ [9]. Here we will show that if we consider the decay of D meson to non-CP final state i.e., $K^{*+}K^-$ for which $r_D \sim \mathcal{O}(1)$, then it is possible to extract the CKM angle γ by measuring only the four decay rates. This method is very promising because the experimental branching ratio for the process $B^0 \rightarrow \bar{D}^0 K^{*0}$ has already been known with value $Br(B^0 \rightarrow \bar{D}^0 K^{*0}) = (5.3 \pm 0.8) \times 10^{-5}$ and for the ($B^0 \rightarrow D^0 K^{*0}$) process we have the upper limit as $Br(B^0 \rightarrow D^0 K^{*0}) < 1.8 \times 10^{-5}$ [9]. The advantage of using the non-CP eigenstate has been discussed in [10], in connection with the charged B decays $B^\pm \rightarrow K^\pm D^0(\bar{D}^0)$, which renders the corresponding interfering amplitudes to be of same order.

Now let us denote the amplitudes for these processes as

$$A_B = \mathcal{A}(\bar{B}^0 \rightarrow D^0 \bar{K}^{*0}), \quad \bar{A}_B = \mathcal{A}(\bar{B}^0 \rightarrow \bar{D}^0 \bar{K}^{*0}), \quad (1)$$

and their ratio as

$$\frac{\bar{A}_B}{A_B} = r_B e^{i(\delta_B - \gamma)}, \quad \text{with} \quad r_B = \left| \frac{\bar{A}_B}{A_B} \right| \quad \text{and} \quad \arg(\bar{A}_B/A_B) = \delta_B - \gamma, \quad (2)$$

where δ_B and $(-\gamma)$ are the relative strong and weak phases between the two amplitudes. The ratio of the corresponding CP conjugate processes are obtained by changing the sign of the weak phase γ . One can then obtain a rough estimate of r_B from dimensional analysis, i.e.,

$$r_B = \left| \frac{V_{ub} V_{cs}^*}{V_{cb} V_{us}^*} \right| \approx 0.4. \quad (3)$$

Now we consider that both D^0 and \bar{D}^0 will decay into the common non-CP final state $(K^{*+} K^-)$. Denoting the D^0 decay amplitudes as

$$A_D = \mathcal{A}(D^0 \rightarrow K^{*+} K^-), \quad \bar{A}_D = \mathcal{A}(\bar{D}^0 \rightarrow K^{*+} K^-), \quad (4)$$

one can write their ratio

$$\frac{\bar{A}_D}{A_D} = r_D e^{i\delta_D}, \quad \text{with} \quad r_D = \left| \frac{\bar{A}_D}{A_D} \right|, \quad (5)$$

where δ_D is the relative strong phase between them. It is interesting to note that the parameters r_D and δ_D have recently been measured by CLEO collaboration [11], with values $r_D = 0.52 \pm 0.05 \pm 0.04$ and $\delta_D = 332^\circ \pm 8^\circ \pm 11^\circ$.

With these definitions the four amplitudes are given as

$$\begin{aligned} \mathcal{A}_1(\bar{B}_d^0 \rightarrow (K^{*+} K^-)_D \bar{K}^{*0}) &= |A_B A_D| \left[1 + r_B r_D e^{i(\delta_B + \delta_D - \gamma)} \right], \\ \mathcal{A}_2(\bar{B}_d^0 \rightarrow (K^{*-} K^+)_D \bar{K}^{*0}) &= |A_B A_D| e^{i\delta_D} \left[r_D + r_B e^{i(\delta_B - \delta_D - \gamma)} \right], \\ \mathcal{A}_3(B_d^0 \rightarrow (K^{*-} K^+)_D K^{*0}) &= |A_B A_D| \left[1 + r_B r_D e^{i(\delta_B + \delta_D + \gamma)} \right], \\ \mathcal{A}_4(B_d^0 \rightarrow (K^{*+} K^-)_D K^{*0}) &= |A_B A_D| e^{i\delta_D} \left[r_D + r_B e^{i(\delta_B - \delta_D + \gamma)} \right]. \end{aligned} \quad (6)$$

From these amplitudes one can obtain the four observables (R_1, \dots, R_4) , with the definition

$$R_i = \left| \mathcal{A}_i(\bar{B}_d^0 \rightarrow (K^{*\pm} K^\mp)_D \bar{K}^{*0}) / (A_B A_D) \right|^2. \quad (7)$$

We can now write $R_1 = 1 + r_B^2 r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D - \gamma)$ and similarly R_2 , R_3 and R_4 . Thus, we get four observables and three unknowns, namely, r_B , δ_B and γ . Hence γ can in principle be determined from these four observables.

Assuming that the amplitudes $|A_B|$ and $|A_D|$ are known (so also r_B , which is expected to be $\sim \mathcal{O}(0.4)$), we obtain an analytical expression for γ as

$$\tan^2 \gamma = \frac{(R_1 - R_3)^2 - (R_2 - R_4)^2}{[R_2 + R_4 - 2(r_B^2 + r_D^2)]^2 - [R_1 + R_3 - 2(1 + r_B^2 r_D^2)]^2}. \quad (8)$$

Thus the measurement of the four observables $R_{1,\dots,4}$ can be used to extract cleanly the CKM angle γ .

Next, we consider the decay channels $B_d^0 \rightarrow D^{*0} K^{*0}$, $\bar{D}^{*0} K^{*0}$ and $\bar{B}_d^0 \rightarrow D^{*0} \bar{K}^{*0}$, $\bar{D}^{*0} \bar{K}^{*0}$ with two vector mesons in the final state. Considering the decay of a B meson into two vector mesons V_1 and V_2 , which subsequently decay into pseudoscalar mesons i.e., $V_1 \rightarrow P_1 P'_1$ and $V_2 \rightarrow P_2 P'_2$, one can write the normalized differential angular distribution as [12],

$$\begin{aligned} \frac{1}{\Gamma} \frac{d^3 \Gamma}{d \cos \theta_1 d \cos \theta_2 d \psi} = \frac{9}{8\pi\Gamma} \left\{ L_1 \cos^2 \theta_1 \cos^2 \theta_2 + \frac{L_2}{2} \sin^2 \theta_1 \sin^2 \theta_2 \cos^2 \psi \right. \\ \left. + \frac{L_3}{2} \sin^2 \theta_1 \sin^2 \theta_2 \sin^2 \psi + \frac{L_4}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \cos \psi \right. \\ \left. - \frac{L_5}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \sin \psi - \frac{L_6}{2} \sin^2 \theta_1 \sin^2 \theta_2 \sin 2\psi \right\}, \quad (9) \end{aligned}$$

where θ_1 (θ_2) is the angle between the three-momentum of P_1 (P_2) in the V_1 (V_2) rest frame and the three-momentum of V_1 (V_2) in the B rest frame, and ψ is the angle between the normals to the planes defined by $P_1 P'_1$ and $P_2 P'_2$, in the B rest frame. The coefficients L_i can be expressed in terms of three independent amplitudes, A_0 , A_{\parallel} and A_{\perp} , which correspond to the different polarization states of the vector mesons as

$$\begin{aligned} L_1 &= |A_0|^2, & L_4 &= \text{Re}[A_{\parallel} A_0^*], \\ L_2 &= |A_{\parallel}|^2, & L_5 &= \text{Im}[A_{\perp} A_0^*], \\ L_3 &= |A_{\perp}|^2, & L_6 &= \text{Im}[A_{\perp} A_{\parallel}^*]. \end{aligned} \quad (10)$$

In the above A_0 , A_{\parallel} , and A_{\perp} are complex amplitudes of the three helicity states in the transversity basis. These observables can be efficiently extracted from the angular distribution (9) using the appropriate weight functions as discussed in Ref. [13].

The decay mode $B \rightarrow V_1 V_2$ can also be described in the helicity basis, where the amplitude for the helicity matrix element can be parameterized as [14]

$$\begin{aligned} H_\lambda &= \langle V_1(\lambda) V_2(\lambda) | \mathcal{H}_{eff} | B^0 \rangle \\ &= \varepsilon_{1\mu}^*(\lambda) \varepsilon_{2\nu}^*(\lambda) \left[a g^{\mu\nu} + \frac{b}{m_1 m_2} p^\mu p^\nu + \frac{ic}{m_1 m_2} \epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} \right], \end{aligned} \quad (11)$$

where p is the B meson momentum, $\lambda = 0, \pm 1$ are the helicity of both the vector mesons and m_i, p_i and ε_i ($i = 1, 2$) denote their masses, momenta and polarization vectors respectively. Furthermore, the three invariant amplitudes a, b , and c are related to the helicity amplitudes by

$$H_{\pm 1} = a \pm c \sqrt{x^2 - 1}, \quad H_0 = -ax - b(x^2 - 1), \quad (12)$$

where $x = (p_1 \cdot p_2)/m_1 m_2 = (m_B^2 - m_1^2 - m_2^2)/(2m_1 m_2)$.

The corresponding decay rate using the helicity basis amplitudes can be given as

$$\Gamma = \frac{p_{cm}}{8\pi m_B^2} \left(|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2 \right), \quad (13)$$

where p_{cm} is the magnitude center-of-mass momentum of the outgoing vector particles.

The amplitudes in transversity and helicity basis are related to each other through the following relations

$$A_\perp = \frac{H_{+1} - H_{-1}}{\sqrt{2}}, \quad A_\parallel = \frac{H_{+1} + H_{-1}}{\sqrt{2}}, \quad A_0 = H_0. \quad (14)$$

The corresponding helicity amplitudes \bar{H}_λ for the complex conjugate decay process $\bar{B} \rightarrow \bar{V}_1 \bar{V}_2$ have the same decomposition with $a \rightarrow \bar{a}, b \rightarrow \bar{b}$ and $c \rightarrow -\bar{c}$. The amplitudes \bar{a}, \bar{b} and \bar{c} can be obtained from a, b and c by changing the sign of the weak phases.

In order to study the feasibility of this method, first we would like to estimate the branching ratios of the above mentioned decay modes. Only, the experimental upper limits for these modes are known so far i.e., $Br(B^0 \rightarrow \bar{D}^{*0} K^*) < 6.9 \times 10^{-5}$ and $Br(B^0 \rightarrow D^{*0} K^*) < 4.0 \times 10^{-5}$ [9]. We expect that these modes will be well measured soon in the currently running asymmetric B factories or in the upcoming LHCb experiment.

In the SM, these decays proceed through color suppressed tree diagrams only and are free from penguin contributions. The decay $B^0 \rightarrow D^{*0} K^{*0}$ arises from the quark level transition $\bar{b} \rightarrow \bar{u} c \bar{s}$ and the process $\bar{B}^0 \rightarrow D^{*0} \bar{K}^{*0}$ arises from $b \rightarrow c \bar{u} s$. To evaluate the hadronic matrix element $\langle O_i \rangle \equiv \langle D^{*0} \bar{K}^{*0} | O_i | \bar{B}_d^0 \rangle$, the factorization approximation has been used.

Thus, in this approach, we obtain the factorized amplitude for the $B_0 \rightarrow D^{*0} K^{*0}$ modes as

$$\begin{aligned}
H &= \frac{G_F}{\sqrt{2}} \lambda_u^* a_2 \langle K^{*0}(\varepsilon_1, p_1) | (\bar{s}b)_{V-A} | B_d^0(p) \rangle \langle D^{*0}(\varepsilon_2, p_2) | (\bar{u}c)_{V-A} | 0 \rangle \\
&= \frac{G_F}{\sqrt{2}} \lambda_u^* a_2 i f_{D^{*0}} m_{D^{*0}} \left[(m_{B^0} + m_{K^{*0}}) A_1^{BK^*}(m_{D^{*0}}^2) (\varepsilon_1^* \cdot \varepsilon_2^*) \right. \\
&\quad \left. - \frac{2 A_2^{BK^*}(m_{D^{*0}}^2)}{(m_{B^0} + m_{K^{*0}})} (\varepsilon_1^* \cdot p) (\varepsilon_2^* \cdot p) - i \frac{2 V^{BK^*}(m_{D^{*0}}^2)}{(m_{B^0} + m_{K^{*0}})} \epsilon_{\mu\nu\alpha\beta} \varepsilon_2^{*\mu} \varepsilon_1^{*\nu} p^\alpha p_1^\beta \right], \quad (15)
\end{aligned}$$

where $f_{D^{*0}}$ is the decay constant of the vector meson D^{*0} and $\lambda_u^* = V_{ub}^* V_{cs}$. Furthermore, $A_1^{BK^*}(m_{D^{*0}}^2)$, $A_2^{BK^*}(m_{D^{*0}}^2)$ and $V^{BK^*}(m_{D^{*0}}^2)$ are the form factors involved in the transition $B^0 \rightarrow K^{*0}$. The coefficients a_2 is given by $a_2 = C_2 + C_1/N_C$, with N_C as the number of colors. Thus, in this way, we can have the invariant amplitudes a , b and c (in the unit of $G_F/\sqrt{2}$) as

$$\begin{aligned}
a &= i a_2 \lambda_u^* f_{D^{*0}} m_{D^{*0}} (m_{B^0} + m_{K^{*0}}) A_1^{BK^*}(m_{D^{*0}}^2), \\
b &= -i a_2 \lambda_u^* f_{D^{*0}} m_{D^{*0}} \frac{2 m_{D^{*0}} m_{K^{*0}}}{(m_{B^0} + m_{K^{*0}})} A_2^{BK^*}(m_{D^{*0}}^2), \\
c &= -i a_2 \lambda_u^* f_{D^{*0}} m_{D^{*0}} \frac{2 m_{D^{*0}} m_{K^{*0}}}{(m_{B^0} + m_{K^{*0}})} V^{BK^*}(m_{D^{*0}}^2). \quad (16)
\end{aligned}$$

Substituting the values of the effective coefficient $a_2 = 0.23$, the Wolfenstein parameters $A = 0.801$, $\lambda = 0.2265$, $\bar{\rho} = 0.189$ and $\bar{\eta} = 0.358$ from [15], the decay constant $f_{D^{*0}} = 240$ MeV, the particle masses and lifetimes from [9] and the form factors $A_1^{BK^*}(m_{D^{*0}}^2) = 0.32$, $A_2^{BK^*}(m_{D^{*0}}^2) = 0.31$ and $V^{BK^*}(m_{D^{*0}}^2) = 0.52$ from [16], we obtain the the branching ratio for the $B^0 \rightarrow D^{*0} K^{*0}$ as

$$Br(B^0 \rightarrow D^{*0} K^{*0}) = 3.87 \times 10^{-6}. \quad (17)$$

Similarly, one can obtain the transition amplitude for the $\bar{B}^0 \rightarrow D^{*0} \bar{K}^{*0}$ process, which is analogous to (15) with the replacement of λ_u^* by $\lambda_c = V_{cb} V_{us}^*$ and hence the corresponding branching ratio as

$$Br(\bar{B}^0 \rightarrow D^{*0} \bar{K}^{*0}) = 2.3 \times 10^{-5}. \quad (18)$$

Since the branching ratios of the above two processes are very much within the reach of the present experiments, we expect that these processes will be measured soon by the currently running B factories and one will have a plenty of such events in the upcoming LHCb factory.

Now, we consider the extraction of $(2\beta + \gamma)$ from the modes $\bar{B}_d^{(-)} \rightarrow \bar{D}^{*0(-)} \bar{K}^{*0(-)}$. Since it is possible to obtain the different helicity contributions by performing an angular analysis

[13, 17], from now onward we will concentrate on the longitudinal (i.e., A_0) component, which is the dominant one. The $K_S\pi^0$ mode of \bar{K}^{*0} allows the $B^0 \rightarrow D^{*0}K^{*0}$ and $\bar{B}^0 \rightarrow D^{*0}\bar{K}^{*0}$ amplitudes to interfere with each other. As discussed earlier, the decay amplitude for the mode $B_d^0 \rightarrow D^{*0}K^{*0}$ arises from $\bar{b} \rightarrow \bar{u}c\bar{s}$ and carries the weak phase $e^{i\gamma}$ while $\bar{B}_d^0 \rightarrow D^{*0}\bar{K}^{*0}$ arises from the quark transition $b \rightarrow c\bar{u}s$ and carries no weak phase. The amplitudes also carry strong phases $e^{i\delta_1}$ and $e^{i\delta_2}$. Thus, we can write the longitudinal components of the decay amplitudes as

$$\begin{aligned} A_0(f) &= \text{Amp}(B_d^0 \rightarrow f)_0 = M_1 e^{i\gamma} e^{i\delta_1}, \\ \bar{A}_0(f) &= \text{Amp}(\bar{B}_d^0 \rightarrow f)_0 = M_2 e^{i\delta_2}, \\ \bar{A}_0(\bar{f}) &= \text{Amp}(\bar{B}_d^0 \rightarrow \bar{f})_0 = M_1 e^{-i\gamma} e^{i\delta_1}, \\ A_0(\bar{f}) &= \text{Amp}(B_d^0 \rightarrow \bar{f})_0 = M_2 e^{i\delta_2}. \end{aligned} \quad (19)$$

Since the final state $f = D^{*0}\bar{K}^{*0}$ is accessible to B^0 and \bar{B}^0 , inserting the time evolution of the observables $A_0(t)$ as in [18], one arrives at the usual expression for the longitudinal component of the time dependent decay widths [19] as

$$\begin{aligned} \Gamma_0(B^0(t) \rightarrow f) &= \frac{e^{-\Gamma t}}{2} \left\{ (|A_0(f)|^2 + |\bar{A}_0(f)|^2) + (|A_0(f)|^2 - |\bar{A}_0(f)|^2) \cos \Delta m t \right. \\ &\quad \left. - 2\text{Im} \left(\frac{q}{p} A_0(f)^* \bar{A}_0(f) \right) \sin \Delta m t \right\}, \\ \Gamma_0(\bar{B}^0(t) \rightarrow f) &= \frac{e^{-\Gamma t}}{2} \left\{ (|A_0(f)|^2 + |\bar{A}_0(f)|^2) - (|A_0(f)|^2 - |\bar{A}_0(f)|^2) \cos \Delta m t \right. \\ &\quad \left. + 2\text{Im} \left(\frac{q}{p} A_0(f)^* \bar{A}_0(f) \right) \sin \Delta m t \right\}, \end{aligned} \quad (20)$$

where $q/p = \exp(-2i\beta)$ is the $B^0 - \bar{B}^0$ mixing parameter and Γ and Δm denote the average width and the mass difference of the heavy and light B mesons and we have neglected the small width difference $\Delta\Gamma$ between them.

Thus, the time dependent measurement of the longitudinal component of $B^0(t) \rightarrow f$ decay rates allow one to obtain the following observables :

$$|A_0(f)|^2 + |\bar{A}_0(f)|^2, \quad |A_0(f)|^2 - |\bar{A}_0(f)|^2, \quad \text{and} \quad \text{Im} \left[\frac{q}{p} A_0(f)^* \bar{A}_0(f) \right], \quad (21)$$

i.e., the longitudinal components of CP averaged branching ratio, the direct CP violation and the mixing induced CP violation parameters.

Similarly one can obtain the time dependent decay rates for the final state \bar{f} i.e., $\Gamma_0(\bar{B}^0(t) \rightarrow \bar{f})$ from $\Gamma_0(B^0(t) \rightarrow f)$ by replacing $A_0(f)$ by $\bar{A}_0(\bar{f})$ and $\bar{A}_0(f)$ by corresponding CP conjugate $A_0(\bar{f})$. $\Gamma_0(B^0(t) \rightarrow \bar{f})$ can be obtained from $\Gamma_0(\bar{B}^0(t) \rightarrow f)$ with similar substitution.

Now substituting the decay amplitudes as defined in Eq. (19) in (20) we get the decay rates as

$$\begin{aligned}
\Gamma_0(B^0(t) \rightarrow f) &= \frac{e^{-\Gamma t}}{2} \left\{ (M_1^2 + M_2^2) + (M_1^2 - M_2^2) \cos \Delta mt \right. \\
&\quad \left. - 2M_1 M_2 \sin(\delta - \phi) \sin \Delta mt \right\}, \\
\Gamma_0(B^0(t) \rightarrow \bar{f}) &= \frac{e^{-\Gamma t}}{2} \left\{ (M_1^2 + M_2^2) - (M_1^2 - M_2^2) \cos \Delta mt \right. \\
&\quad \left. + 2M_1 M_2 \sin(\delta + \phi) \sin \Delta mt \right\}, \\
\Gamma_0(\bar{B}^0(t) \rightarrow \bar{f}) &= \frac{e^{-\Gamma t}}{2} \left\{ (M_1^2 + M_2^2) + (M_1^2 - M_2^2) \cos \Delta mt \right. \\
&\quad \left. - 2M_1 M_2 \sin(\delta + \phi) \sin \Delta mt \right\}, \\
\Gamma_0(\bar{B}^0(t) \rightarrow f) &= \frac{e^{-\Gamma t}}{2} \left\{ (M_1^2 + M_2^2) - (M_1^2 - M_2^2) \cos \Delta mt \right. \\
&\quad \left. + 2M_1 M_2 \sin(\delta - \phi) \sin \Delta mt \right\}, \tag{22}
\end{aligned}$$

where $\delta = \delta_2 - \delta_1$ is the strong phase difference between the longitudinal components of the two amplitudes $\bar{B}^0 \rightarrow f$ and $B^0 \rightarrow f$ and $\phi = 2\beta + \gamma$. Thus through the measurements of the time dependent rates, it is possible to measure the amplitudes M_1 and M_2 and the CP violating quantities $S_+ \equiv \sin(\delta + \phi)$ and $S_- \equiv \sin(\delta - \phi)$. In turn these quantities will determine $\tan^2 \phi$ up to a four fold ambiguity via the expression

$$\tan^2 \phi [\cot^2 \delta] = \frac{(S_+ - S_-)^2}{2 - S_-^2 - S_+^2 \pm 2\sqrt{(1 - S_+^2)(1 - S_-^2)}}, \tag{23}$$

where one sign will give $\tan^2 \phi$ and the other $\cot^2 \delta$.

Let us now estimate the number of reconstructed events that could be observed at the B factories assuming that 3×10^8 (10^{12}) $B\bar{B}$'s are (will be) available at the e^+e^- asymmetric B factories (hadronic B machines like LHCb). Let us first estimate the number of $B^0 \rightarrow D^0(\bar{D}^0)K^{*0}$ events will be available in the upcoming LHCb experiment. Assuming the branching ratio for the process $B^0 \rightarrow D^0 K^{*0}$ to be $|(V_{ub}V_{cs}^*)/(V_{cb}V_{us}^*)|^2 \times Br(B^0 \rightarrow \bar{D}^0 K^{*0}) \approx$

8.5×10^{-6} , $Br(D^0 \rightarrow K^{*+}K^-) = 3.7 \times 10^{-3}$ [9] and 10% overall reconstruction efficiency, we expect to get nearly 9×10^3 events per year of running at LHCb. For the corresponding vector-vector modes, we use the longitudinal component of the branching ratio as $Br_0(B^0 \rightarrow D^{*0}K^{*0}) = 0.65 \times Br(B^0 \rightarrow D^{*0}K^{*0}) \approx 2.51 \times 10^{-6}$, $Br(D^{*0} \rightarrow D^0\pi^0) = 62\%$ [9], $Br(K^{*0} \rightarrow K_S\pi^0) = Br(K^{*0} \rightarrow K\pi)/3$, and an overall efficiency of 10%. Thus we expect to get approximately 15 (5×10^4) reconstructed events at the e^+e^- (hadronic) machines per year of running. This crude estimate indicates that this method may be well suited for the extraction of the weak phase $\gamma(2\beta + \gamma)$ at LHCb.

In this paper, we have carried out a study of the color suppressed decay modes $B^0 \rightarrow D^0(D^{*0})K^{*0}$ to extract the weak phase $\gamma(2\beta + \gamma)$. For the extraction of γ we considered the decay modes $B^0 \rightarrow D^0(\bar{D}^0)K^{*0}$, with subsequent decay of $D^0(\bar{D}^0)$ into non-CP state $K^{*+}K^-$. The use of the non-CP state allows the two interfering amplitudes to be of same order and hence one can cleanly extract the CKM angle γ . Next we considered the processes $B^0 \rightarrow D^{*0}(\bar{D}^{*0})K^{*0}$, where the final states are admixtures of CP-even and CP-odd states. However, it is possible to disentangle them using the angular distributions of the final decay products. Now considering the longitudinal component of the time-dependent decay rates of these modes we have shown that $\phi \equiv (2\beta + \gamma)$ can be cleanly obtained. Since these modes are free from penguin pollution and also the branching ratios are measurable at hadron factories such as the LHCb, we feel that they could be very much suited for determining the phase $\gamma(2\beta + \gamma)$.

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